# LC 2015: Paper 1

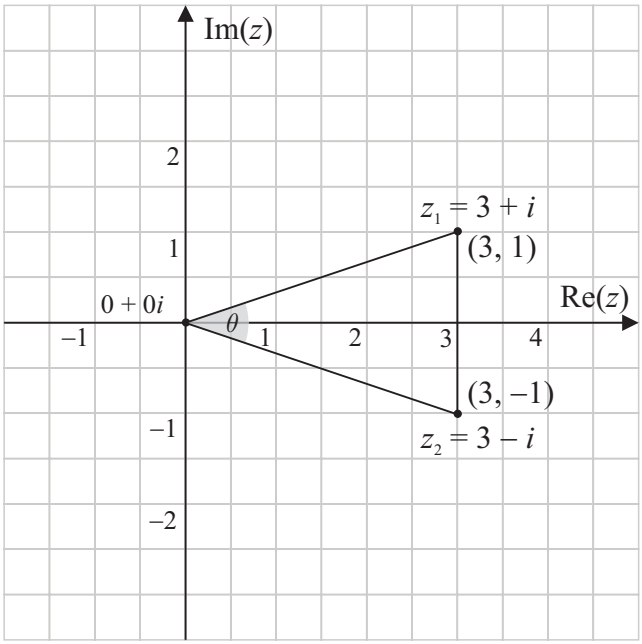
## Question 6 (25 marks)

### Question 6 (a)

**Complex numbers**
$a + bi = c + di$
Equate the real parts: $a = c$
Equate the imaginary parts: $b = d$

(i) $z_1 = 3 + i = a + bi \Rightarrow a = 3,\ b = 1$

(ii) $z_2 = 3 - i = c + di \Rightarrow c = 3,\ d = -1$



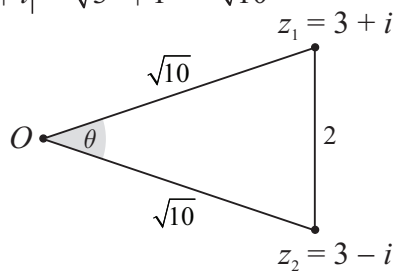

**Marking Scheme Notes**
**Question 6 (a) [Scale 15D (0, 3, 7, 11, 15)]**
**3**: • One correct value
• Plots $z_2$ correctly and stops
**7**: • Two correct values
**11**: • Three correct values

### Question 6 (b) (i)

Find the modulus of each complex number. This is the distance from the origin (0, 0) to each complex number.

$|z_1| = |3 + i| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$|z_2| = |3 + i| = \sqrt{3^2 + 1^2} = \sqrt{10}$




Apply the Cosine rule as you are given 3 sides in the triangle.

$2^2 = (\sqrt{10})^2 + (\sqrt{10})^2 - 2(\sqrt{10})(\sqrt{10})\cos\theta$

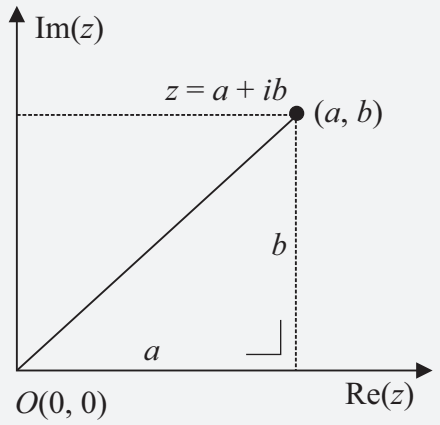
$4 = 10 + 10 - 20\cos\theta$

$4 = 20 - 20\cos\theta$

$20\cos\theta = 16$

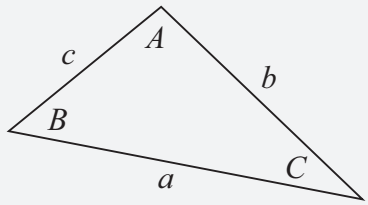
$\therefore \cos\theta = \frac{16}{20} = \frac{4}{5} = 0\cdot 8$

**Formula: Complex Numbers**
Modulus $|z|$




$|z| = |a + bi| = \sqrt{a^2 + b^2}$

**Formulae and Tables Book**
**Trigonometry of the triangle:**
**Cosine rule** [page 16]




$a^2 = b^2 + c^2 - 2bc\cos A$

**Marking Scheme Notes**
**Question 6 (b) (i) [Scale 5B (0, 3, 5)]**
**3**: • Cosine formula written
• Finds $|z_1|$
• Any work of merit
• Joins $oz_1$ or $oz_2$ or $z_1z_2$

**Question 6 (b) (ii)**

Show the left-hand side (**LHS**) is equal to the right-hand side (**RHS**).

| **LHS** | **RHS** |
|---|---|
| $\lvert z_1\rvert\times\lvert z_2\rvert\times\cos\theta$ | $ac+bd$ |
| $=\sqrt{10}\sqrt{10}(0\cdot 8)$ | $=(3)(3)+(1)(-1)$ |
| $=8$ | $=9-1$ |
| | $=8$ |

$\therefore \lvert z_1\rvert\times\lvert z_2\rvert\times\cos\theta = ac+bd$

**Marking Scheme Notes**
**Question 6 (b) (ii) [Scale 5C (0, 3, 4, 5)]**
**3**: • $ac + bd$ correct
• Any work of merit
• Finds $|z_1|$ and/or $|z_2|$
**7**: • one error only in substitution, with conclusion